Olley and Pakes-style production function estimators with firm fixed effects

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Abstract

We show that control function estimators (CFEs) of the firm production function, such as Olley-Pakes, may be biased when productivity evolves with a firm-specific drift, in which case the correctly specified control function will contain a firm-specific term, omitted in the standard CFEs. We develop an estimator that is free from this bias by introducing firm fixed effects in the control function. Applying our estimator to the data, we find that it outperforms the existing CFEs in terms of capturing persistent unobserved heterogeneity in firm productivity. Our estimator involves minimal modification to the standard CFE procedures and can be easily implemented using common statistical software.

Keywords: production function; control function estimator; panel data.

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1 Introduction

Much of the empirical literature on firm-level productivity relies on estimating the production function. A well-known problem in this literature is what Griliches and Mairesse (1998) call the *transmission bias* – a bias in input elasticity estimates caused by a correlation between factor inputs and unobserved firm productivity. Olley and Pakes (1996) introduced a control function estimator (CFE) which has become a popular solution to this bias. Their approach was to control for the correlation between factor inputs and unobserved firm productivity by proxying the latter with a function of observed firm characteristics that reflect a firm's reaction to productivity changes.

Several studies have since emerged, extending the classical Olley-Pakes CFE (CFE-OP) estimator to address its limitations. However, the available CFEs still rely on a number of assumptions which are rarely tested. Among those is the assumption that total factor productivity (TFP) follows a first-order Markov process that is homogeneous for all firms; that is, $\omega_{it} = E[\omega_{it}|\omega_{it-1}] + v_{it}$, where ω is the TFP, v is the i.i.d. innovation term, and i and t are firm and time indicators, respectively. In this paper, we extend this assumption to include a firm-specific unobserved heterogeneity term η_i, specifying $\omega_{it} = E[\omega_{it}|\omega_{it-1}, \eta_i] + v_{it}$, micro-found this extension, and show that the conventional CFEs will be inconsistent unless all $\eta_i = 0$. We then derive a CFE with firm fixed effects (CFE-FE) that controls for $\eta_i \neq 0$ and apply it to two widely-used data sets.

Our work is motivated by the fact that, while substantial and persistent productivity differences between firms have long been seen in the data (Bartelsman and Doms, 2000; Syverson, 2011), the available CFEs lack controls for firm heterogeneity in TFP. This deficiency, which has been recognized as one of the serious limitations of the CFE approach (Eberhardt and Helmers, 2010), may cause transmission bias in the production function estimates. Especially prone to this bias is the coefficient on capital input, since capital is likely to be more highly correlated with the persistent productivity component than other inputs because capital adjustment costs are higher.[1] Another motivation for our effort to incorporate firm heterogeneity in the CFE framework is the observation from Aw (2002), as well as from our data, that firm productivity only gradually converges to its steady-state level, and that this level varies by firm. The productivity dynamics we observe are consistent with a Markov process with a firm-specific drift; yet, the conventional CFEs assume the productivity specification without a drift.

We derive a set of moment conditions for a consistent CFE under the specification of TFP extended to include a firm-specific drift. The resulting estimator, CFE-FE, differs from the conventional CFEs in that it controls for firm fixed effects in the estimation procedure, which we micro-found. Specifically, applying a selection of the conventional CFEs to the manufacturing firm data from Denmark and Chile, we find a significant and lasting autocorrelation in their regression residuals. We attribute this autocorrelation to the presence of a persistent TFP component that the control function and the TFP specification with $\eta_i = 0$ failed to capture. This residual autocorrelation is greatly reduced when we apply the CFE-FE, implying that our estimator captures a large part of the firm-specific persistence in productivity. An additional advantage of our estimator is its conceptual simplicity and ease of implementation: in particular, it can be run using existing Stata commands (.do files available from the authors).

In the rest of the paper, we outline the existing CFEs starting with CFE-OP (Sections 2.1-2.2), and show how unaccounted persistence in the TFP leads to the transmission bias (section 2.3). Then, in Section 3, we explain how introducing fixed effects in the CFE framework can mitigate this bias and provide details on our new CFE-FE. We apply a selection of CFEs to the Chilean and Danish manufacturing firm data in

[1] Since labor or material input adjustment costs are relatively low, their response to short-term productivity shocks will be higher than that of capital.

section 4, showing in particular the large difference in residual persistence between the existing CFEs and CFE-FE. Section 5 concludes.

2 Control function-based estimators (CFEs)

Consider a Cobb-Douglas production function (in logs):

$$\begin{aligned} va_{it} &= l_{it}\beta_l + k_{it}\beta_k + u_{it} \\ u_{it} &= \omega_{it} + e_{1,it}, \end{aligned} \tag{1}$$

where i, t are the firm and year indicators, respectively, va_{it} is value added, l_{it} is the vector of *static* inputs, such as labor, which can vary freely at each t, k_{it} is a vector of *dynamic* inputs, such as capital, which are partly determined by their previous stock. The term u_{it}, unobservable to the econometrician, is the empirical equivalent of Hicks-neutral productivity. It is the sum of total factor productivity (TFP) ω_{it}, which is observed by the firm and hence affects its input choices, and random noise $e_{1,it}$, which is unobserved and does not affect input choices. Though ω_{it} can be correlated with l_{it} or k_{it}, we restrict $e_{1,it}$ to be orthogonal to l_{it}, k_{it}, and ω_{it}.

Estimating equation (1) with OLS will result in biased and inconsistent estimates of β_l and β_k because a firm's choice of input quantities depends on its TFP: more productive firms will use more, resulting in the transmission bias mentioned in the introduction. One approach to deal with such bias is to proxy TFP with a function of observables, called a *control function*. This section gives an overview of existing control function estimators (CFEs) and outlines the key assumptions required for their consistency, showing that some of them are quite fragile. The next section presents our proposed estimator that can be a solution when some of these assumptions fail to hold.

2.1 The Olley-Pakes estimator (CFE-OP)

The Olley and Pakes (1996) CFE-OP estimator deals with the endogeneity problem arising from the correlation between input factors and TFP ω_{it} by using a control function of observables that carry information on ω_{it}. The estimation procedure relies on several assumptions common to other CFEs as we outline below. The original CFE-OP contains a correction for potentially endogenous firm entry and exit, which can be implemented in our setting if necessary. For the sake of simplicity, we do not implement this correction in this paper, focussing instead on the main issue of interest – the control function.

Assumption 1 *k_{it} at time t is predetermined, while l_{it} is freely adjustable for each t.*

The second part of Assumption 1, regarding the scope for adjusting the labor input l_{it} as well as other static inputs, is relaxed in later modifications of the CFE-OP which we review in the next subsection.

Assumption 2 *("Scalar unobservability") The investment function i_{it} is fully determined by the dynamic inputs k_{it}, the TFP ω_{it}, and, possibly, other observable variables z_{it}.*

Under Assumptions 1 and 2, the firm investment level that solves the dynamic profit maximisation problem can be represented as a function of the state variables (k_{it}, z_{it}) and TFP:

$$i_{it} = \phi\left(\omega_{it}, k_{it}, z_{it}\right) \tag{2}$$

Assumption 3 *The investment function $i_{it} = \phi(\omega_{it}, k_{it}, z_{it})$ in (2) is monotonic in ω_{it}.*

Assumption 3 implies that the control function can be specified by inverting the investment function (2) for ω_{it}:

$$\omega_{it} = g\left(k_{it}, i_{it}, z_{it}\right) \tag{3}$$

Putting the control function (3) back in the production function (1) gives the first-stage CFE-OP regression:

$$va_{it} = l_{it}\beta_l + k_{it}\beta_k + g\left(k_{it}, i_{it}, z_{it}\right) + e_{1,it} \tag{4}$$

The unknown function $g\left(\cdot\right)$ is approximated with a polynomial of a fixed (usually third) order in (k_{it}, i_{it}, z_{it}), denoted $\widetilde{g}(\cdot)$. [2] Approximating $g\left(\cdot\right)$ with $\widetilde{g}(\cdot)$ does not allow β_k to be estimated from (4) but does allow the recovery of the coefficient estimates for static factor inputs, $\widehat{\beta}_l$, and the estimated composite term $\widehat{\Phi}_{it}$ that captures the TFP and the dynamic inputs,

$$\Phi_{it} \equiv \Phi\left(k_{it}, i_{it}, z_{it}\right) = k_{it}\beta_k + g\left(k_{it}, i_{it}, z_{it}\right), \tag{5}$$

from the following first-stage regression:

$$va_{it} = l_{it}\beta_l + \Phi\left(k_{it}, i_{it}, z_{it}\right) + e_{1,it}. \tag{6}$$

Estimating β_k requires an additional assumption on TFP:

Assumption 4 *ω_{it} follows the first-order Markov process: $\omega_{it} = E\left[\omega_{it}|\omega_{it-1}\right] + e_{2,it}$, where $e_{2,it}$ is an innovation term satisfying $E\left[e_{2,it}|\omega_{it-1}\right] = 0$.*

Under Assumption 4, for a given β_l, it follows from (1) that

$$E\left[va_{it} - l_{it}\beta_l|k_{it}, \omega_{it-1}\right] = k_{it}\beta_k + E\left[\omega_{it}|k_{it}, \omega_{it-1}\right] + E\left[e_{1,it} + e_{2,it}|k_{it}, \omega_{it-1}\right]$$

and $E\left[e_{1,it} + e_{2,it}|k_{it}, \omega_{it-1}\right] = 0$ by construction, since k_{it} is pre-determined from As-

[2] As a robustness check, we have also tried a second- and a fourth-degree polynomials, both giving very similar regression estimates and diagnostic statistics. In particular, the shares in the residual variance accounted by the third- and fourth-degree polynomials in $(k_{it}; i_{it}; z_{it})$ are within 0.005 from each other.

sumption 1 and $e_{1,it}$ is orthogonal to $(l_{it}, k_{it}, \omega_{it})$. The regression

$$va_{it} - l_{it}\beta_l = k_{it}\beta_k + E\left[\omega_{it}|\omega_{it-1}\right] + e_{it}$$

with $e_{it} = e_{1,it} + e_{2,it}$ is then well-defined and has no endogeneity problem. Letting $E\left[\omega_{it}|\omega_{it-1}\right] = h\left(\omega_{it-1}\right)$ and noting that $\omega_{it-1} = \Phi_{it-1} - k_{it-1}\beta_k$ from (3) and (5), β_k can be consistently estimated from the second stage regression

$$va_{it} - l_{it}\widehat{\beta}_l = k_{it}\beta_k + h\left(\widehat{\Phi}_{it-1} - k_{it-1}\beta_k\right) + e_{it}, \tag{7}$$

with the unknown function $h\left(\cdot\right)$ again approximated by a fixed-order polynomial and $\widehat{\beta}_l$ and $\widehat{\Phi}_{it-1}$ obtained from the first-stage regression (6).

2.2 Extensions of the Olley-Pakes estimator

Several limitations of CFE-OP have been identified since it was introduced, motivating other CFEs as its extensions. One such limitation is that in practice many firms report zero investment, which casts doubt on the monotonicity of the investment function (Assumption 3). In particular, the presence of capital adjustment costs could violate the monotonicity assumption, making the investment function non-invertible. To address this concern, Levinsohn and Petrin (2003) include intermediate inputs, such as materials which are always positive, in the control function. Thus, the proxy for productivity in their CFE-LP estimator is

$$\omega_{it} = g(k_{it}, m_{it}, z_{it}), \tag{8}$$

where m_{it} is log materials input. Like investments, materials are chosen optimally by firms given the state variables, but the adjustment costs of materials are arguably lower than of capital investment, so that the monotonicity assumption is less likely to fail.

Another potential problem with CFE-OP and CFE-LP, discussed in Ackerberg, Caves, and Frazer (2015), is that the coefficient on labor input may not be identifiable at the first stage. This problem will arise if labor input is optimally chosen by firms upon observing their productivity, in which case it becomes a function $l_{it} = \varphi(k_{it}, z_{it}, \omega_{it})$. Substituting the expression for ω_{it} from equation (8), $l_{it} = \varphi(k_{it}, z_{it}, g(k_{it}, m_{it}, z_{it}))$. That is, labor input becomes a function of the same variables as the control function, which precludes identification of its coefficient, β_l, at the first stage. The solution proposed by Ackerberg, Caves, and Frazer (2015) is to estimate β_l from the second stage, using the estimate of the control function from the first stage and Assumption 4. Their procedure amounts to estimating the following (nonlinear) regression equation:

$$va_{it} = l_{it}\beta_l + k_{it}\beta_k + h\left(\widehat{\Phi}_{it-1} - l_{it-1}\beta_l - k_{it-1}\beta_k\right) + e_{it}, \tag{9}$$

which differs from the standard procedure in equation (7) in that the coefficients on capital and labor are estimated together. Wooldridge (2009) proposes a GMM procedure that estimates all the coefficients in the production function in one stage by directly approximating the function $h(.)$ in (9) with a polynomial in (k, l, m, z). This estimator is somewhat simpler to implement because it is linear and does not rely on the estimates of $\Phi(.)$ from the first stage, thus avoiding bootstrapping to compute standard errors. The linearity of Wooldridge (2009) estimator is particularly important for our purposes because running a nonlinear estimator with high-dimension fixed effects may be computationally challenging.

In our estimation procedure, we combine the choice of the instruments for the control function in CFE-OP and CFE-LP with the flexibility of the Wooldridge (2009) GMM estimator. This combination results in what we label as the CFE-WOP estimator by running

$$va_{it} = l_{it}\beta_l + k_{it}\beta_k + f\left(k_{it-1}, m_{it-1}, z_{it-1}\right) + e_{it} \tag{10}$$

with the moment conditions

$$E\left(e_{it}|k_{it}, l_{it-1}, k_{it-1}, m_{it-1}, z_{it-1}\right) = 0, \tag{11}$$

where m is log materials input, and $f(\cdot)$ is approximated by a polynomial function in $(k_{it-1}, m_{it-1}, z_{it-1})$.[3] We adopt this CFE-WOP as the baseline estimator in this study.

Yet another potential issue with CFE-OP and CFE-LP is the fragility of the scalar unobservability Assumption 2, the failure of which results in inconsistent estimates. To illustrate, consider CFE-OP and define $r_{it} = g(k_{it}, i_{it}, z_{it}) - \widetilde{g}(k_{it}, i_{it}, z_{it})$ as the difference between TFP $\omega_{it} = g(k_{it}, i_{it}, z_{it})$ and its polynomial approximation $\widetilde{g}(k_{it}, i_{it}, z_{it})$. In the standard case, r_{it} will be a part of the regression residuals and can be made arbitrarily small by increasing the degree of the polynomial in $\widetilde{g}(\cdot)$. However, if we omit important variables in the investment function $\phi\left(\cdot\right)$ (and hence in the control function), r_{it} will be a function of the omitted variables. In this case r_{it} cannot be made arbitrarily small by increasing the order of polynomial approximation. Huang and Hu (2011) develop a maximum likelihood estimator that is robust to the presence of measurement error in the proxy variable (or, equivalently, omitted variables in z_{it}). Their solution is to use another proxy variable that is independent of the original proxy conditional on unobserved productivity. In their estimation procedure, one proxy variable works as an instrument for the other, producing unbiased control function estimates. Their estimator can outperform the alternatives, such as Ackerberg, Caves, and Frazer (2015), in the presence of measurement error in the proxy variable. This type of non-degenerating

[3] Alternatively, one can estimate the same specification with gross output as the dependent variable, in which case one should use instruments other than the current materials input in the control function (for example, investment).

approximation error problem can also happen when Assumption 4 is violated, which we further explore in the next section.

2.3 Persistence in the TFP process and its consequences

In this section we describe another situation in which existing CFEs will give inconsistent estimates: the presence of firm-specific persistence in TFP. Suppose TFP follows a first-order Markov process conditional on a random variable η_i with a finite second moment and $E[e_{1,it}|\eta_i] = 0$,

$$\omega_{it} = E\left[\omega_{it}|\omega_{it-1}, \eta_i\right] + e_{2,it}, \tag{12}$$

where $e_{2,it}$ satisfies $E\left[e_{2,it}|\omega_{it-1}, \eta_i\right] = 0$. The specification of TFP in (12) is a generalization of the first-order Markov condition in Assumption 4 underlying CFE-OP. The existing CFEs all assume homogeneous dynamics in TFP, that is, $\eta_i = 0$. If in fact $\eta_i \neq 0$, the consequence of this assumption is a misspecified second-stage CFE regression (7) which restricts $\eta_i = 0$ or, equivalently, misspecified moment conditions (11) which omit η_i. These misspecifications will result in inconsistent CFE estimates if factor inputs or proxy variables are correlated with η_i.

Firm-specific persistence in productivity, which is consistent with specification (12), can be observed in the data. For illustrative purposes, take a simple example of (12), a first-order stationary autoregressive process with a firm-specific drift,

$$E\left[\omega_{it}|\omega_{it-1}, \eta_i\right] = \eta_i + \gamma\omega_{it-1} \tag{13}$$

with $0 < \gamma < 1$. The above specification implies that each firm has its steady-state productivity level, $\eta_i/(1-\gamma)$, that it gradually approaches. The distinct statistical "signature" of such a specification is slower productivity growth in older firms which are closer to their steady-state productivity. Aw (2002) finds this pattern in firm data from Taiwan, observing that productivity grows faster in already more productive firms, but this accelerating productivity growth slows down with firm size. As more productive firms tend to be larger, the finding of productivity growth decelerating with size implies a gradual convergence of firm productivity to its steady-state level.

We observe similar productivity dynamics in our data, too. Figure 1 reports average productivity by firm age in the Chilean and Danish manufacturing sectors, for three measures: average value added per worker in all firms of a given age (Panel A), the same adjusted for firm survival probability (Panel B), and the same calculated relative to the average productivity of all firms in a given year (Panel C). For all three measures, productivity grows faster in newer firms and slows down as firms get older and bigger. A first-order Markov process without a drift (Assumption 4) would not generate such productivity patterns, because then the average of the firm productivity levels would fluctuate around some stable level without positive dynamics for newer firms. Moreover, breaking down the sample of market entrants by terciles of TFP in the first year of entry (Figure 2), we find the same dynamic pattern in all terciles, but the productivity levels in different terciles converge to distinctively different values. Hence, not only is there a drift in TFP but also this drift is firm-specific.

[Figures 1, 2 here.]

3 The CFE with firm fixed effects

In this section, we develop a new CFE estimator with firm fixed effects, labelled CFE-FE. Unlike other CFEs, CFE-FE is consistent in the presence of firm-specific persistence in productivity as specified in (12).

3.1 CFE-FE: The consistent estimator under firm-specific persistence in TFP

The estimator we present in this section is based on Assumptions 1-3 and the following generalization of Assumption 4 (recall the TFP specification in equation (12)):

Assumption 4′ *TFP ω_{it} follows the first-order Markov process conditional on a random variable η_i with a finite second moment:* $\omega_{it} = E\left[\omega_{it}|\omega_{it-1}, \eta_i\right] + e_{2,it}$, *where* $E\left[e_{2,it}|\omega_{it-1}, \eta_i\right] = 0$ *and* $E[e_{1,it}|\eta_i] = 0$.

Unlike the standard CFE approaches, which all assume homogeneous dynamics in the TFP, the TFP specification in Assumption 4′ allows for firm-specific persistence in TFP through the term η_i. We do not impose any restrictions on the statistical relation between ω_{it-1} and η_i; hence they can be arbitrarily correlated with each other. In this sense, we can see η_i as the *fixed effect* in panel data models.

We consider a particular version of (12) where $E\left[\omega_{it}|\omega_{it-1}, \eta_i\right] = \eta_i + h(\omega_{it-1})$ for some unknown function $h(\cdot)$. This way we separate the persistent TFP component, η_i, from the rest of it. This specification yields a nonparametric panel autoregression with fixed effects:

$$\omega_{it} = \eta_i + h(\omega_{it-1}) + e_{2,it}. \tag{14}$$

Under the above TFP specification, the first step of the estimation procedure is the same as in other CFEs except for adding the firm fixed effect η_i. One can still use the first-stage regression (6) in order to estimate β_l and $\Phi\left(\cdot\right)$. Under Assumptions 1-3 and Assumption 4′, at the second stage we have

$$E\left[va_{it} - l_{it}\hat{\beta}_l|k_{it}, \omega_{it-1}, \eta_i\right] = k_{it}\beta_k + E\left[\omega_{it}|k_{it}, \omega_{it-1}, \eta_i\right] + E\left[e_{1,it} + e_{2,it}|k_{it}, \omega_{it-1}, \eta_i\right]$$

for a given $\widehat{\beta}_l$ obtained at the first stage, where $E\left[e_{1,it} + e_{2,it}|k_{it}, \omega_{it-1}, \eta_i\right] = 0$. By letting $E\left[\omega_{it}|\omega_{it-1},\eta_i\right]=\eta_i+h(\omega_{it-1})$ as in (14), the regression

$$\begin{aligned} va_{it}-l_{it}\widehat{\beta}_l &= k_{it}\beta_k+E\left[\omega_{it}|\omega_{it-1},\eta_i\right]+e_{it} \\ &= \eta_i+k_{it}\beta_k+h(\omega_{it-1})+e_{it} \end{aligned}$$

with $e_{it}=e_{1,it}+e_{2,it}$ has no endogeneity problem. Noting that $\omega_{it-1}=\Phi_{it-1}-k_{it-1}\beta_k$, we can consistently estimate β_k from a semiparametric panel regression with a firm-specific fixed effect at the second stage:

$$va_{it}-l_{it}\widehat{\beta}_l=\eta_i+k_{it}\beta_k+h(\widehat{\Phi}_{it-1}-k_{it-1}\beta_k)+e_{it} \tag{15}$$

with the unknown $h(\cdot)$ approximated by a (fixed-order) polynomial and $\widehat{\beta}_l$ and $\widehat{\Phi}_{it-1}$ obtained from the first stage.

Alternatively, in the spirit of Wooldridge (2009), all the parameters in the above equation can be estimated in one stage, by running

$$va_{it}=\eta_i+l_{it}\beta_l+k_{it}\beta_k+f(k_{it-1},m_{it-1},z_{it-1})+e_{it}, \tag{16}$$

which is the approach we take. As noted in Ahn and Schmidt (1995), Arellano and Bover (1995) and Blundell and Bond (1998), we can estimate the coefficients β_l,β_k, as well as the parameters of the polynomial approximation $f(k_{it-1},m_{it-1},z_{it-1})$, from the following moment conditions:

$$E[\Delta x_{is}(e_{it}+\eta_i)]=0 \quad \text{for } s\leq t-1 \text{ and } t=2,\cdots,T, \tag{17}$$

where $x_{is}=(l_{is},k_{is+1},m_{is},z_{is+1})'$. The moment conditions in (17) are valid when x_{is} is stationary over t, because under stationarity $E\left[\Delta x_{is}\eta_i\right]=0$.

Remark 1 Our estimation approach involves estimating equation (16) in levels using the first-differenced instruments in the moment conditions (17). As an alternative, one can estimate equation (16) in first differences with moment conditions (17) in levels,

$$E\left[\Delta e_{it} | \{(l_{is}, k_{is+1}, m_{is}, z_{is+1})\}_{s \leq t-2}\right] = 0, \tag{18}$$

or combine the moment conditions in (17) and (18) in the same GMM estimator. Under the usual assumptions, the moment conditions (18) are valid even for non-stationary instruments. However, this approach should be used with caution in the presence of measurement error in factor inputs, because the first-differencing of equation (16) will greatly increase the share of measurement error in the regressors, rendering the estimates inconsistent. On the other hand, our approach, while not immune to the bias due to measurement error, will not exacerbate this bias by first-differencing. A more complete treatment of measurement error within our estimation framework is left for further research.

Remark 2 We can add a fixed effect μ_i in the first-stage regression (6) as well, which could control for firm heterogeneity in the production function, some measurement error in inputs, or some omitted (time-invariant) factors in the production function that the productivity fixed effect η_i does not capture. Doing so will require running our estimator in two steps, estimating

$$va_{it} = l_{it}\beta_l + \Phi\left(k_{it}, m_{it}, z_{it}\right) + \mu_i + e_{1,it} \tag{19}$$

at the first step. Since μ_i can be arbitrarily correlated with $(l_{it}, k_{it}, \omega_{it})$ or $(l_{it}, k_{it}, m_{it}, z_{it})$, adding it amounts to extending the scalar unobservability Assumption 2 to include persistent unobservables. The moment conditions for estimating (19) remain the same.

3.2 Comparison of CFE-FE with the dynamic panel estimators with fixed effects

The CFE-FE is related to the dynamic panel estimators with firm fixed effects. Under (14) and the conditions specified in Lee (2014), TFP can be represented as a stationary β-mixing process conditional on η_i. Specifically, ω_{it} can be rewritten as a combination of persistent (time-invariant) and transient (time-varying) elements, $\omega_{it} = F(\eta_i, \{e_{2,it-j}\}_{j=0}^{\infty})$, for some function $F(\cdot,\cdot)$, in which the persistent component of TFP is represented by η_i and the transient component is represented by a combination of $\{e_{2,it-j}\}_{j=0}^{\infty}$. For instance, assuming a linear stationary autoregressive process for TFP as in (13), we obtain

$$\omega_{it} = \eta_i + \gamma\omega_{it-1} + e_{2,it} = \frac{\eta_i}{1-\gamma} + \sum_{j=1}^{\infty} \gamma^j e_{2,it-j}.$$

In the above expression, TFP is a simple sum of the persistent component $b_i = \eta_i/(1-\gamma)$ and the transient component $a_{it} = \sum_{j=1}^{\infty} \gamma^j e_{2,it-j}$.

In fact, in this linear specification, CFE-FE can be rewritten as the dynamic panel regression as Blundell and Bond (1998, 2000):

$$va_{it} = \eta_i + \gamma va_{it-1} + (l_{it} - \gamma l_{it-1})\beta_l + (k_{it} - \gamma k_{it-1})\beta_k + \xi_{it}$$

with $\xi_{it} = (u_{it} - \gamma u_{it-1}) + e_{2,it}$. Both the CFE-FE and dynamic panel estimators such as those in Blundell and Bond (1998, 2000) will yield the same results when they share the same initial conditions under (14) and the conditions in Lee (2014). However, this identity will only hold when $E[\omega_{it}|\omega_{it-1}, \eta_i]$ is linear as in (13). When $E[\omega_{it}|\omega_{it-1}, \eta_i]$ is nonlinear, the dynamic panel approach will result in inconsistent estimates. Since our approach is nonparametric in $E[\omega_{it}|\omega_{it-1}, \eta_i]$, similar to the original CFE-OP, it will yield consistent estimates even under nonlinear Markov dynamics in ω_{it}.

As an illustration, consider $E[\omega_{it}|\omega_{it-1},\eta_i] = \eta_i + \gamma_1\omega_{it-1} + \gamma_2\omega_{it-1}^2$. Though $E[\omega_{it}|\omega_{it-1},\eta_i]$ is still linear in parameters, this quadratic specification cannot be fully controlled in the dynamic panel regression above, hence the regression error term becomes $\xi_{it} = \gamma_2\omega_{it-1}^2 + (u_{it} - \gamma_1 u_{it-1}) + e_{2,it}$. Then, since ω_{it-1}^2 can be correlated with va_{it-1}, l_{it-1}, or k_{it-1}, the dynamic panel estimators will not be consistent without more exogenous instruments.

3.3 Heterogeneous investment response to TFP components

One of the benefits of the framework from which we have derived CFE-FE (Section 3.1) is that we can extend it to allow investment to respond differently to the persistent and transient TFP components. The existing CFEs assume that investment response to a given change in TFP will be the same regardless of whether it is caused by persistent or short-term components. This assumption may not be true, in which case the moment conditions (11) can be violated. For example, financial constraints that new firms typically face will lengthen the accumulation of their capital stock to the optimal level given their productivity (Moll, 2014). Then, the investment it takes to build up the required capital will be little affected by short-term productivity shocks along the transition path towards the optimal scale. Factor adjustment costs further dampen the reaction of a firm to short-term productivity shocks. As Cooper and Haltiwanger (2006) show, capital adjustment costs mute the response of firm investment to productivity shocks with fast mean reversion as compared to more persistent productivity shocks.

Allowing for firm fixed effects in the control function as in Assumption $4'$ can resolve this problem. For instance, consider the following investment function:

$$i_{it} = \phi\left([a_{it} + \delta b_i], k_{it}, z_{it}\right) = \phi\left([\omega_{it} + (\delta - 1)b_i], k_{it}, z_{it}\right), \tag{20}$$

where $0 < \delta \neq 1$ is set to allow for investment responses to b_i and to a_{it} to be different. Inverting (20) for $[\omega_{it} + (\delta - 1)b_i]$ gives the control function

$$\omega_{it} = g\left(k_{it}, i_{it}, z_{it}\right) + (1 - \delta)b_i, \tag{21}$$

which one can approximate in the usual way but with added firm fixed effects at the first stage and then proceed to the second stage. Hence, if Assumptions 1-3 and 4$'$ are satisfied, a CFE with a fixed effect in the control function will produce consistent estimates. We leave the more general case of investment being an arbitrary function of a_{it} and b_i, as well as micro-foundation of this case, for further research.

4 Empirical results

In this section we apply a selection of existing estimators to two popular datasets in the firm productivity literature to show that they fail to account for persistence in firm productivity, relegating part of it to the error term. We also demonstrate that our estimators absorb much of this persistence, thus reducing the potential transmission bias that cannot be controlled by conventional CFEs. We report the results from both value added and gross output specifications of the production function.

4.1 Data

The first dataset comes from *Instituto Nacional de Estadistica* and covers all Chilean manufacturing plants with more than 10 employees during the years between 1979-1996. These data have been used in many studies of firm-level productivity, including Levinsohn and Petrin (2003), Gandhi, Navarro, and Rivers (2011), and Ackerberg, Caves, and Frazer (2015), as well as in applications of productivity analysis in other contexts, most notably Pavcnik (2002), Kasahara and Rodrigue (2008), and Petrin and Levinsohn (2013). For each of the 10,927 plants in our sample, the data include the four-digit ISIC industry code identifier, gross output, material inputs, capital stock and investments, and labor input measured in person-years, converted where necessary into real values using industry price deflators. A more detailed description of the data is available in Levinsohn and Petrin (2003).

The second dataset comes from the Danish Business Statistics Register, maintained by Statistics Denmark, and includes all firms registered in Denmark. In this study we use information on manufacturing firms for the years 1995-2007. The variables observed in each year include the four-digit NACE industry identifier, total output, value added, fixed assets, investments, material inputs, and employment (headcount measure). Lentz and Mortensen (2008), Munch and Skaksen (2008), Stoyanov and Zubanov (2012), and de Loecker and Warzynski (2012) used these data.

4.2 Estimation results

Tables 1 and 2 present the OLS and the standard control function estimation results of the production function for Chile and Denmark, respectively. In each table, column "OLS" shows the linear regression results of (1); column "OLS-FE" shows the estimation results of linear regression of (1) with fixed effect (i.e., the within-group estimator); column "OP" shows the standard control function estimation results of Olley and Pakes (1996) or Levinsohn and Petrin (2003); and column "WOP" shows the estimation results using the Wooldridge (2009) GMM approach.[4] The rest of the columns present the WOP results for the three largest manufacturing industries in each country. The standard errors in all specifications are estimated from the residuals clustered at the firm level.

Tables 1 and 2 also report the first-order autoregressive coefficient estimates of the regression residuals: ρ_1 is that of the first-stage regression residuals from (6), and ρ_2

[4]We use the Stata command `ivreg2` for the GMM estimation, after approximating the unknown functions by polynomials.

is that of the second-stage regression residuals from (7). We compute the autocorrelation coefficients based on our estimates of the first- and second-stage residuals. The former are estimated from equation (4) which gives the first-stage residuals $e_{1,it}$ plus the approximation error. We compute the second-stage residuals $e_{2,it}$ as the difference between the total residual from (7) and first-stage residuals. Although our estimates of the residuals are imperfect because of the presence of approximation error, the residual autocorrelation coefficients calculated from them are still informative.

TABLE 1 HERE.

TABLE 2 HERE.

Tables 1 and 2 show that the overall residual autocorrelation coefficient ρ in the OLS and OP regressions without fixed effects is above 0.5. Furthermore, as seen in the correlogram in Figure 3, the high residual autocorrelation hardly declines with time. An interesting case is the OLS regression with fixed effects in column "OLS-FE", where the residual autocorrelation is much lower: 0.088 for the Danish and 0.241 for the Chilean samples. The comparison between the regressions with and without firm fixed effects, as well the stability of residual autocorrelation over time, hints at the presence of a persistent component in TFP.

FIGURE 3 HERE.

The coefficients ρ_1, ρ_2 reveal a strong autocorrelation in the first-stage, and a weaker but still considerable autocorrelation in the second-stage residuals. The high values of ρ_1 (0.54 and higher) imply the presence of a persistent component in the TFP that the control function has not captured, which is in breach of the scalar unobservability Assumption 3. Significant autocorrelation in the second-stage residuals implies the presence of a firm-specific drift in the TFP process, contradicting Assumption 4. Thus, the estimation results so far suggests that a CFE with firm fixed effects in the control function and in the TFP process is required.

Another potential source of persistence in the CFE residuals may be a higher-order Markov process in TFP – a threat to the consistency of CFE other than a firm-specific persistent TFP component. The presence of a higher-order Markov process in productivity can be examined and addressed by extending the existing CFEs to allow for a second-order Markov process,

$$\omega_{it} = \mathrm{E}\left[\omega_{it}|\omega_{it-1}, \omega_{it-2}\right] + e_{2,it},$$

at the cost of extra assumptions on the control function and at least one additional proxy variable, besides materials, to separately identify the two lags of ω_{it} (Ackerberg, Lanier Benkard, Berry, and Pakes, 2007; Stoyanov and Zubanov, 2014). We implement the estimator proposed in these studies by choosing capital investment as the additional proxy.

TABLE 3 HERE.

Table 3 reports the regression estimates and residual autocorrelations for the value added specification with firm productivity following a second-order Markov process without firm fixed effects. This modification produces second-stage residuals $e_{2,it}$ that are less strongly autocorrelated than those from the standard CFE. However, the first-stage residual autocorrelation is essentially unaffected in both the Chile and Denmark samples, implying that missing higher-order Markov terms are unlikely to be an important factor contributing to residual persistence. Its most likely source remains to be the presence of firm-specific persistence in TFP, which our estimator is capable of addressing.

Table 4 shows the factor input elasticities using parameter estimates from the CFE-FE for the Chilean and Danish samples and, in square brackets, their differences with the respective estimates from the conventional CFE (columns 4 to 7 in Tables 1 and

2). The table also reports the first-order autoregression coefficient estimates of the regression residuals, ρ_1 and ρ_2, as in Tables 1 and 2.

[TABLE 4 HERE]

The results in Table 4 support the inclusion of firm fixed effects by showing a considerable reduction in the magnitude of residual autocorrelation coefficients as compared to the standard CFE estimates in Tables 1 and 2: a range of 0.106-0.289 for Chile and 0.006-0.075 for Denmark in the CFE specifications with fixed effects versus 0.5-0.6 without. The omitted factors which were the sources of residual autocorrelation do not seem to be random effects in the error terms $e_{1,it}$ or $e_{2,it}$ since the input elasticity estimates change once fixed effects are introduced. The changes in the input elasticity estimates reflect correlations between persistent TFP heterogeneity, omitted in the conventional CFEs, and factor inputs. For example, in the value added specification estimated on the Danish sample, the capital input elasticity estimate goes down by two-fifth, or by 7.5 standard errors, and the estimate of labor coefficient goes up by one-fourth, or by 10 standard errors. In the gross output specification, it is the coefficients on labor and material inputs that change the most as compared to their CFE-WOP estimates.

5 Conclusion

In this paper, we have identified a potential transmission bias in the production function estimators based on the control function approach. This bias occurs because the control function does not fully capture productivity persistence. We identify and micro-found a case when this happens: when productivity follows a dynamic process with a firm-specific effect. We show that this case can be dealt with by introducing firm fixed effects in the control function and derive a consistent estimator, the CFE-FE. We also extend the CFE-FE framework to address the case when investment (or materials) responds differently to transient and persistent productivity components, and show that CFE-FE is consistent under the assumptions we outline.

We show our estimator in action by reporting a substantial firm-specific component in the regression residuals estimated from data on Chilean and Danish manufacturing firms. The presence of this component is a marker of the transmission bias, since a correctly specified control function would absorb all relevant firm heterogeneity. We then show that applying our estimator greatly reduces persistence in the residuals. Importantly to applied researchers, our estimator easy to implement. An approximation of the control function allows for an uncomplicated GMM procedure which can be implemented using the Stata `ivreg2` command.

The advantages of CFE-FE notwithstanding, allowing for fixed effects can exacerbate the attenuation bias due to measurement error (Griliches and Hausman, 1986), which poses a potential tradeoff between the transmission bias in the conventional CFEs and the attenuation bias in CFE-FE. Addressing this tradeoff is left for further research. Currently, there is a case for caution in using our estimator when the persistent TFP component is weak and the extent of measurement error in the data is large.

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Table 1. Production function estimation results from existing estimators, Chile.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Estimator	OLS	OLS-FE	OP	WOP	WOP	WOP	WOP
Industry sample	All manufacturing industries				Food products	Fabricated metals	Textiles
					(311)	(381)	(321)
	Dependent variable: log value added						
Labor	1.023	0.813	0.689	0.758	0.582	0.795	0.623
	(0.018)	(0.016)	(0.017)	(0.017)	(0.027)	(0.045)	(0.047)
Capital	0.308	0.128	0.387	0.349	0.321	0.221	0.259
	(0.010)	(0.009)	(0.018)	(0.018)	(0.035)	(0.056)	(0.048)
N	54,801	54,801	49,228	49,265	14,823	4,171	4,189
ρ	0.696	0.264	0.644	0.647	0.491	0.569	0.576
	(0.003)	(0.007)	(0.007)	(0.007)	(0.014)	(0.025)	(0.024)
$\rho1$			0.625	0.637	0.496	0.535	0.540
			(0.006)	(0.007)	(0.014)	(0.024)	(0.023)
$\rho2$			-0.222	-0.042	0.155	-0.046	0.171
			(0.008)	(0.016)	(0.020)	(0.074)	(0.029)
	The share of between-firm variation in residuals						
e1	0.57	n/a	0.51	0.53	0.37	0.47	0.46
e2			0.16	0.19	0.11	0.24	0.11
	Dependent variable: log output						
Labor	0.301	0.285	0.246	0.243	0.159	0.286	0.233
	(0.008)	(0.008)	(0.010)	(0.009)	(0.011)	(0.025)	(0.021)
Capital	0.088	0.044	0.198	0.113	0.057	0.111	0.087
	(0.004)	(0.004)	(0.045)	(0.009)	(0.015)	(0.030)	(0.023)
Materials	0.722	0.645	0.714	0.740	0.866	0.687	0.766
	(0.005)	(0.007)	(0.007)	(0.006)	(0.008)	(0.019)	(0.015)
N	54,857	54,801	49,228	49,265	14,823	4,171	4,189
ρ	0.644	0.241	0.708	0.702	0.570	0.620	0.577
	(0.003)	(0.008)	(0.006)	(0.013)	(0.032)	(0.028)	(0.032)
$\rho1$			0.679	0.705	0.539	0.648	0.547
			(0.012)	(0.015)	(0.037)	(0.031)	(0.039)
$\rho2$			-0.275	0.136	0.344	0.376	0.102
			(0.013)	(0.016)	(0.037)	(0.073)	(0.065)
	The share of between-firm variation in residuals						
e1	0.53	n/a	0.60	0.64	0.48	0.56	0.54
e2			0.21	0.35	0.58	0.38	0.34

Note: Standard errors in parentheses are clustered by firm. WOP stands for the Wooldridge (2009) modification of the OP estimator. For OP and WOP estimators, the share of between-firm variation in residuals is calculated for the first-stage regression.

Table 2. Production function estimation results from existing estimators, Denmark.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Estimator	OLS	OLS-FE	OP	WOP	WOP	WOP	WOP
Industry sample	All manufacturing industries				Fabricated metals	Printing	Food products
					(28)	(22)	(15)
	Dependent variable: log value added						
Labor	0.917	0.723	0.631	0.658	0.724	0.686	0.655
	(0.004)	(0.003)	(0.006)	(0.005)	(0.010)	(0.014)	(0.012)
Capital	0.139	0.108	0.203	0.102	0.094	0.079	0.105
	(0.003)	(0.003)	(0.002)	(0.003)	(0.005)	(0.007)	(0.008)
N	151,556	151,556	122,938	122,902	24,235	15,061	14,890
ρ	0.777	0.136	0.648	0.554	0.528	0.554	0.535
	(0.002)	(0.008)	(0.007)	(0.008)	(0.015)	(0.019)	(0.031)
ρ1			0.559	0.582	0.550	0.554	0.541
			(0.006)	(0.007)	(0.015)	(0.015)	(0.022)
ρ2			-0.143	-0.212	-0.163	-0.180	-0.164
			(0.005)	(0.007)	(0.014)	(0.028)	(0.020)
	The share of between-firm variation in residuals						
e1	0.75	n/a	0.56	0.56	0.51	0.55	0.52
e2			0.26	0.21	0.32	0.20	0.29
	Dependent variable: log output						
Labor	0.436	0.476	0.425	0.352	0.441	0.478	0.312
	(0.006)	(0.002)	(0.006)	(0.005)	(0.009)	(0.015)	(0.006)
Capital	0.055	0.059	0.031	0.019	0.025	0.017	0.019
	(0.002)	(0.002)	(0.001)	(0.002)	(0.003)	(0.005)	(0.004)
Materials	0.527	0.388	0.521	0.601	0.516	0.542	0.682
	(0.005)	(0.008)	(0.005)	(0.004)	(0.008)	(0.012)	(0.005)
N	151,504	151,504	123,082	123,082	23,880	15,027	14,887
ρ	0.593	0.088	0.581	0.559	0.577	0.496	0.555
	(0.003)	(0.011)	(0.008)	(0.009)	(0.018)	(0.021)	(0.023)
ρ1			0.572	0.566	0.580	0.534	0.553
			(0.007)	(0.008)	(0.017)	(0.017)	(0.023)
ρ2			-0.206	-0.184	-0.167	-0.088	-0.185
			(0.005)	(0.005)	(0.014)	(0.018)	(0.015)
	The share of between-firm variation in residuals						
e1	0.57	n/a	0.56	0.54	0.55	0.51	0.47
e2			0.24	0.29	0.31	0.25	0.24

Note: Standard errors in parentheses are clustered by firm. WOP stands for the Wooldridge (2009) modification of the OP estimator. For OP and WOP estimators, the share of between-firm variation in residuals is calculated for the first-stage regression.

Table 3. Estimation results from the benchmark CFE-OP with a second-order Markov process in TFP.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Chile				Denmark			
	All	Food products	Fabricated metals	Textiles	All	Fabricated metals	Printing	Food products
		(311)	(381)	(321)		(28)	(22)	(15)
Labor	0.504	0.425	0.594	0.527	0.683	0.738	0.714	0.656
	(0.010)	(0.020)	(0.029)	(0.031)	(0.006)	(0.012)	(0.017)	(0.013)
	[-0.254]	[-0.157]	[-0.201]	[-0.096]	[0.025]	[0.014]	[0.028]	[0.001]
Capital	0.366	0.314	0.442	0.231	0.080	0.069	0.066	0.093
	(0.022)	(0.045)	(0.067)	(0.058)	(0.003)	(0.006)	(0.007)	(0.008)
	[0.017]	[-0.007]	[0.211]	[-0.028]	[-0.022]	[-0.025]	[-0.013]	[-0.012]
N	16,081	4,127	1,503	1,261	83,948	16,604	10,080	9,857
ρ	0.730	0.553	0.356	0.588	0.577	0.554	0.579	0.534
	(0.011)	(0.025)	(0.058)	(0.040)	(0.008)	(0.016)	(0.020)	(0.025)
$\rho 1$	0.707	0.519	0.616	0.589	0.575	0.553	0.577	0.558
	(0.012)	(0.028)	(0.033)	(0.041)	(0.008)	(0.017)	(0.018)	(0.024)
$\rho 2$	-0.066	-0.173	-0.026	-0.051	-0.125	-0.164	-0.058	-0.117
	(0.016)	(0.019)	(0.088)	(0.030)	(0.012)	(0.019)	(0.020)	(0.027)
	The share of between-firm variation in residuals							
e1	0.63	0.47	0.56	0.55	0.55	0.51	0.53	0.54
e2	0.17	0.10	0.26	0.17	0.24	0.19	0.28	0.26

Note: Standard errors in parentheses are clustered by firm. For OP and WOP estimators, the share of between-firm variation in residuals is calculated for the first-stage regression.

Table 4. Production function estimation results from WOP with fixed effects

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Chile				Denmark			
	All industries	Food products (311)	Fabricated metals (381)	Textiles (321)	All industries	Fabricated metals (28)	Printing (22)	Food products (15)
	Dependent variable: log value added							
Labor	0.739	0.551	0.732	0.709	0.841	0.850	0.912	0.810
	(0.020)	(0.050)	(0.067)	(0.071)	(0.017)	(0.024)	(0.034)	(0.045)
	[-0.019]	[-0.031]	[-0.063]	[0.086]	[0.183]	[0.126]	[0.226]	[0.155]
Capital	0.360	0.359	0.366	0.301	0.071	0.060	0.065	0.098
	(0.020)	(0.037)	(0.074)	(0.048)	(0.004)	(0.007)	(0.009)	(0.010)
	[0.011]	[0.038]	[0.145]	[0.042]	[-0.031]	[-0.034]	[-0.014]	[-0.007]
N	44,063	13,285	3,302	4,000	101,248	19,354	12,217	12,066
ρ	0.298	0.255	0.258	0.219	0.059	0.061	0.022	0.034
	(0.008)	(0.014)	(0.030)	(0.028)	(0.010)	(0.019)	(0.023)	(0.026)
ρ1	0.296	0.227	0.215	0.251	0.057	0.061	0.029	0.054
	(0.009)	(0.014)	(0.029)	(0.032)	(0.011)	(0.019)	(0.022)	(0.027)
ρ2	0.012	0.040	0.199	0.008	-0.162	-0.081	0.080	-0.050
	(0.037)	(0.028)	(0.071)	(0.101)	(0.012)	(0.026)	(0.042)	(0.027)
	Dependent variable: log output							
Labor	0.185	0.131	0.288	0.238	0.496	0.493	0.544	0.282
	(0.017)	(0.023)	(0.040)	(0.047)	(0.018)	(0.023)	(0.031)	(0.023)
	[-0.058]	[-0.028]	[0.002]	[0.005]	[0.144]	[0.052]	[0.066]	[-0.030]
Capital	0.121	0.073	0.048	0.090	0.027	0.020	0.055	0.027
	(0.015)	(0.024)	(0.054)	(0.039)	(0.003)	(0.005)	(0.003)	(0.006)
	[0.008]	[0.016]	[-0.063]	[0.003]	[0.008]	[-0.005]	[0.038]	[0.008]
Materials	0.762	0.855	0.688	0.737	0.453	0.461	0.389	0.683
	(0.014)	(0.022)	(0.034)	(0.037)	(0.015)	(0.020)	(0.024)	(0.018)
	[0.022]	[-0.011]	[0.001]	[-0.029]	[-0.148]	[-0.055]	[-0.153]	[0.001]
N	17,683	4,540	1,464	1,322	101,377	19,381	12,224	12,077
ρ	0.204	0.161	0.226	0.106	0.006	0.05	0.075	0.044
	(0.017)	(0.043)	(0.057)	(0.045)	(0.001)	(0.030)	(0.030)	(0.031)
ρ1	0.202	0.135	0.148	0.089	0.064	0.055	0.071	0.048
	(0.017)	(0.049)	(0.055)	(0.046)	(0.012)	(0.029)	(0.030)	(0.031)
ρ2	0.037	0.054	0.021	0.052	-0.115	-0.049	0.080	-0.073
	(0.026)	(0.069)	(0.034)	(0.048)	(0.009)	(0.014)	(0.024)	(0.031)

Note: Standard errors in parentheses are clustered by firm. For OP and WOP estimators, the share of between-firm variation in residuals is calculated for the first-stage regression

Figure 1. Value added per worker of new firms.

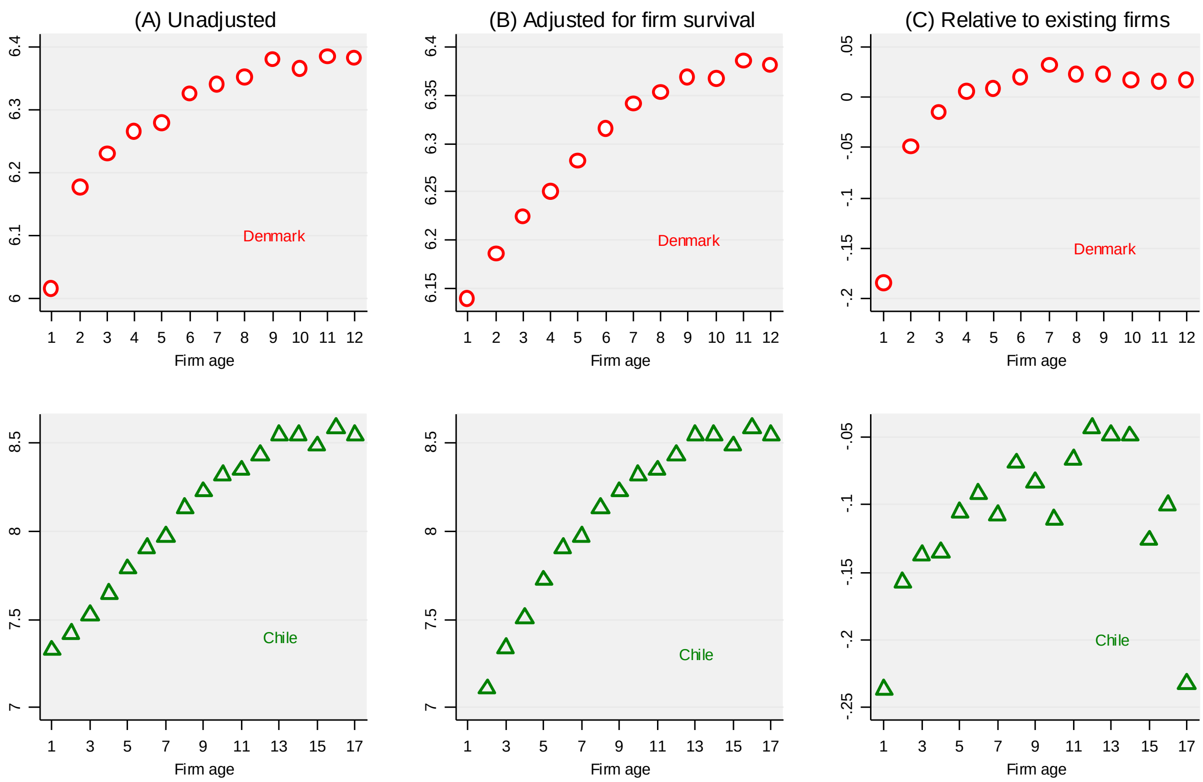


Figure 2. Value added per worker of new firms, by tercile in the year of entry.

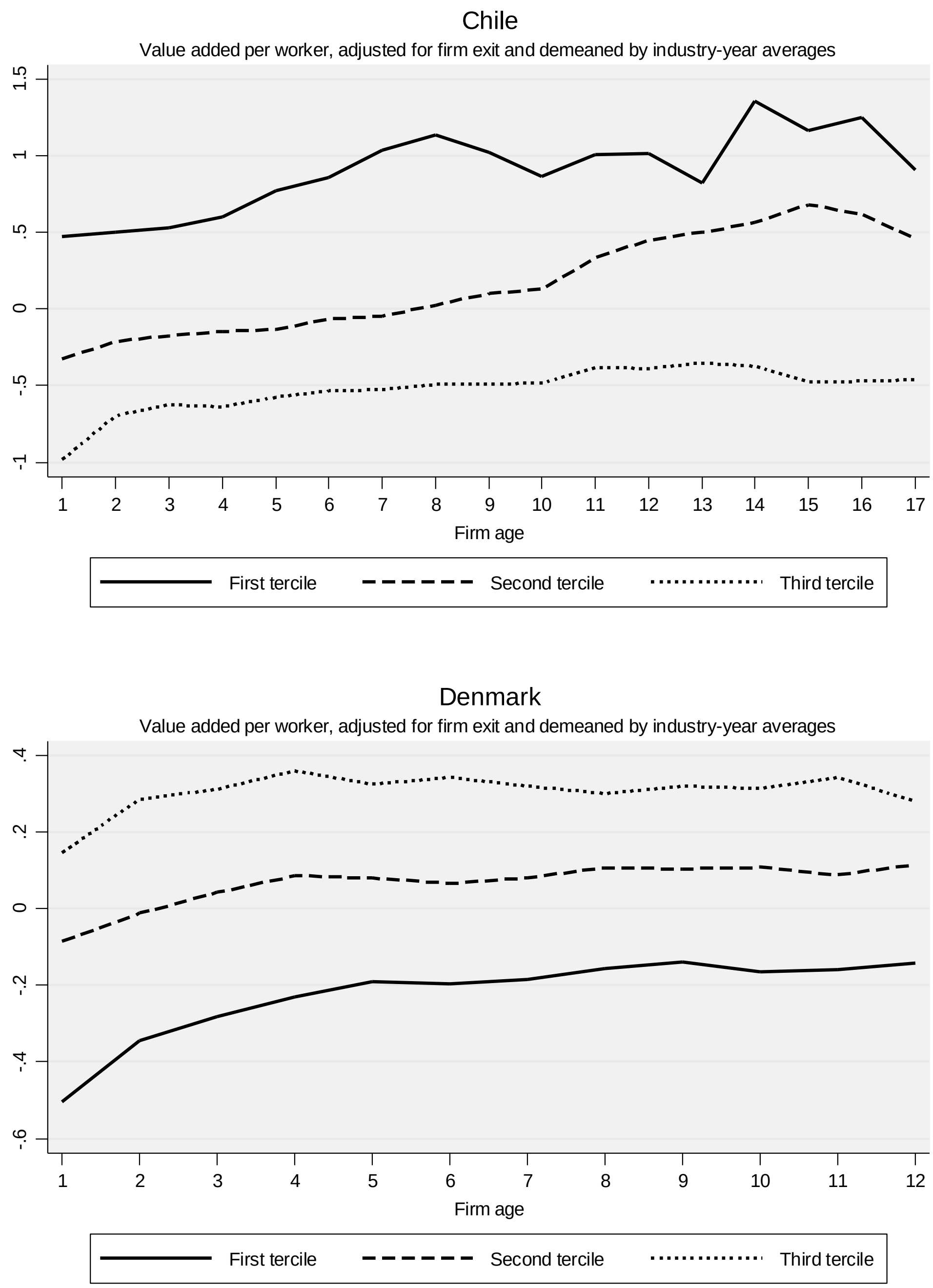
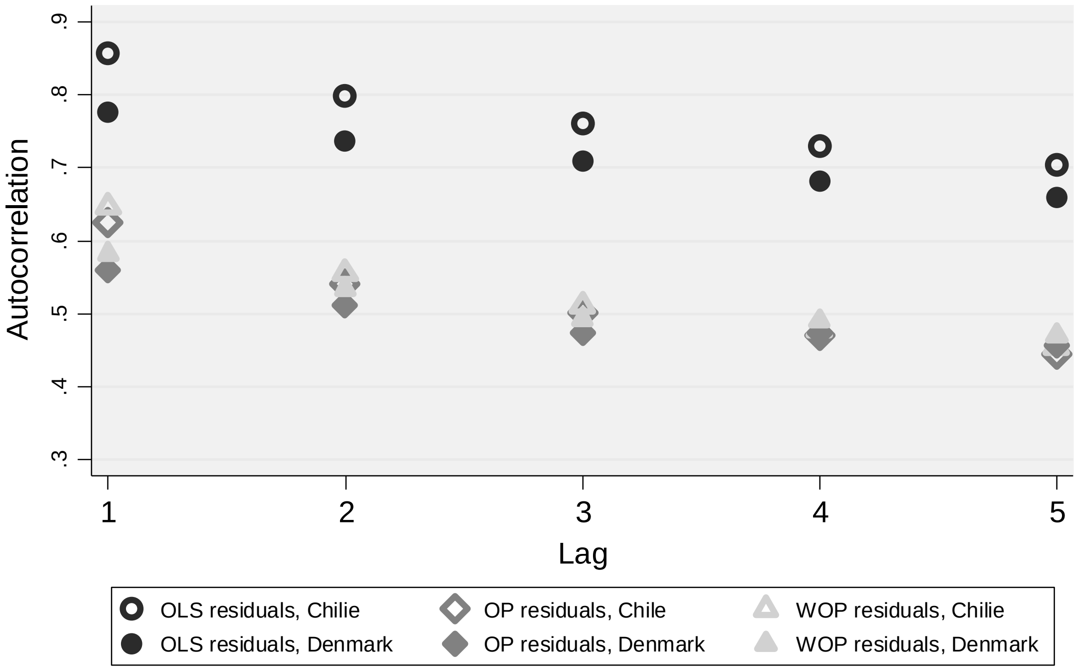


Figure 3. Autocorrelation correlogram for the regression residuals.



Note: total residuals are used for OLS; first-stage residuals for OP and WOP